PRECALC Class ex.   Sec. 6.8: Trig Equations II

1) Solve each trig equation over the interval $0 \le \theta < 2\pi$.

a) $\cos(2\theta) + 3 = 5\cos\theta$ $\qquad \underline{\theta = \frac{\pi}{3}, \frac{5\pi}{3}}$

$(2\cos^2\theta - 1) + 3 = 5\cos\theta$ $\qquad\qquad (2u-1)(u-2) = 0$

$2\cos^2\theta - 5\cos\theta + 2 = 0$ $\qquad\qquad (2\cos\theta - 1)(\cos\theta - 2) = 0$

let $u = \cos\theta$ $\quad 2u^2 - 5u + 2 = 0$ $\qquad\quad \underline{2\cos\theta = 1} \quad \cos\theta = 2 \to NS$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \frac{}{2} \quad \frac{}{2}$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \cos\theta = \frac{1}{2} \to \theta = \frac{\pi}{3}, \frac{5\pi}{3}$

b) $\cos^2\theta + \sin\theta = 2$ $\qquad \underline{\text{NO Solution}}$

$1 - \sin^2\theta + \sin\theta = 2$ $\qquad\qquad\qquad 0 = u^2 - u + 1$

$0 = \sin^2\theta - \sin\theta - 1 + 2$ $\qquad\qquad$ Discriminant $= b^2 - 4ac$

$0 = \sin^2\theta - \sin\theta + 1$ $\qquad\qquad\qquad\qquad\qquad = (-1)^2 - 4(1)(1)$

let $u = \sin\theta$ $\qquad\qquad\qquad\qquad\qquad\qquad\qquad = -3 < 0,$ thus

c) $\sin\theta\cos\theta = -\frac{1}{2}$ $\qquad \underline{\theta = \frac{3\pi}{4}, \frac{7\pi}{4}}$ $\qquad\qquad$ No real Solution

$2 \cdot \sin\theta\cos\theta = \frac{-1}{2} \cdot 2$ $\qquad\qquad \sin u = -1 \to u = \frac{3\pi}{2} + 2k\pi$

$2\sin\theta\cos\theta = -1$ $\qquad\qquad\qquad \frac{2\theta}{2} = \frac{\frac{3\pi}{2}}{2} + \frac{2k\pi}{2}$

$\sin(2\theta) = -1$

let $u = 2\theta$ $\qquad\qquad\qquad\qquad \theta = \frac{3\pi}{4} + k\pi \qquad k=0 \to \theta = \frac{3\pi}{4}$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad k=1 \to \theta = \frac{7\pi}{4}$

d) $\sin\theta + \cos\theta = 1$ $\qquad \underline{\theta = 0, \frac{\pi}{2}}$

① since you can't sub $\qquad\quad (\sin\theta + \cos\theta)^2 = (1)^2$

in any identities, try $\qquad\quad \sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta = 1$

squaring each side.

$\qquad\qquad\qquad\qquad\qquad 1 + 2\sin\theta\cos\theta = 1 \to \frac{2\sin\theta\cos\theta}{2} = \frac{0}{2}$

$\qquad\qquad\qquad\qquad\qquad \sin\theta\cos\theta = 0$

$\qquad\qquad\qquad\qquad\qquad \sin\theta = 0 \qquad\qquad\qquad \cos\theta = 0$

② Sub All 4 solution's into original $\qquad \boxed{\theta = 0, \pi} \qquad\qquad \boxed{\theta = \frac{\pi}{2}, \frac{3\pi}{2}}$

+ you'll see $\theta = \pi, \frac{3\pi}{2}$ are extraneous

2. Use a graphing calculator to solve $5\sin x + x = 3$. Express the solutions rounded to two decimal places.

$\qquad\qquad\qquad \underline{x = 0.52, 3.18, 5.71}$

$Y_1 = 5\sin x + x$ $\qquad$ • use 7: Zoom Trig & change x max to 5π

$Y_2 = 3$ $\qquad\qquad\qquad$ • use 2nd Trace $\to$ 5: intersect to find 3

$\qquad\qquad\qquad\qquad\qquad$ intersection points